

CS 440: Programming Languages

Assignment: Axiomatic Semantics

Logistics and Submission

Please submit your solutions as a PDF (typed or *neatly* handwritten!) on Blackboard by the due date.

Hoare Logic

1. [10 points] Consider the following IMP program, which we will refer to as **C**:

```
x := y - z;

if x < 5 then
  x := y * z
else
  x := y - z
```

Derive the weakest precondition **P** such that the Hoare triple $\{ \mathbf{P} \} \mathbf{C} \{ x > 0 \}$ is valid. You must show your work in steps, deriving the weakest assertion required before each step of the program. Use relational, logical, and arithmetic operators as needed.

2. [10 points] Consider the following IMP program:

```
i := 0;
j := len(arr) - 1;
while i <= j do
  if arr[i] <= arr[0] then
    i := i + 1
  else
    tmp := arr[i];
    arr[i] := arr[j];
    arr[j] := tmp;
    j := j - 1
```

Given a non-empty array of integers **arr** (with indexes 0 through $\text{len}(\text{arr}) - 1$), the program attempts to *partition* **arr** such that all elements lesser than or equal to its first element appear before all elements greater than its first element.

Come up with assertions **P** (the loop invariant) and **Q** (the loop postcondition), in terms of **i**, **j**, and **arr**, that can be used to demonstrate partial correctness of the program. Briefly explain how **P** can be used to imply **Q**.