Q1.
x:=y-z;
if(x<5) then
    *x:=y*z;*
else
    x:=y-z;

Solution:

We can calculate **x:=y-z;** and **x:=y*z**; with Q≡{x>0} first, we obtain:

1. {y*z>0} x:=y*z {x>0}
2. {y-z>0} x:=y-z; {x>0}

Now we calculate the weakest precondition of **the if rule**. Based on 1.{y-z>0} and 2.{y*z>0}, we obtain

3. {y*z>0 Λ x<5} V {y-z>0 Λ x>=5} the if statement {x>0}

Then, we calculate the weakest precondition of **the *x:=y-z;*** *Based on 3.* {y*z>0 Λ x<5} V {y-z>0 Λ x>=5}

4. {(y*z>0 Λ y-z<5) V (y-z>0 Λ y-z>=5)} *x:=y-z;* {y*z>0 Λ x<5} V {y-z>0 Λ x>=5}

Finally, we have:

5. {y*z>0 Λ y-z<5} V {y-z>0 Λ y-z>=5} C {x>0}

Which equivalent to **P≡{y*z>0 Λ y-z<5} V {y-z>=5}**

Q2.

```
i := 0;
j := len(arr) - 1;
while i <= j :
    if arr[i] <= arr[0]:
        i:=i+1;
    else:
        tmp := arr[i];
        arr[i] := arr[j];
        arr[j] := tmp;
        j:=j-1;
```

Solution:

The postcondition is a rigorous restatement of the loop goal, which says all elements arr[0 .. j] are less than or equal to arr[0], and all elements arr[j + 1 .. len(arr)-1] are greater than arr[0].

$$\mathrm{Q} = \forall\, \mathrm{x}\, (0 \leq \mathrm{x} \leq \mathrm{j} \longrightarrow \mathrm{arr}[\mathrm{x}] \leq \mathrm{arr}[0]) \wedge \forall \mathrm{y}\, (\mathrm{j} < \mathrm{y} < \mathrm{len}(\mathrm{arr}) \longrightarrow \mathrm{arr}[\mathrm{y}] > \mathrm{arr}[0])$$

The loop invariant is based on a partial partitioning of the array.

$$\mathrm{P} = \forall\, \mathrm{x}\, (0 \leq \mathrm{x} < \mathrm{i} \longrightarrow \mathrm{arr}[\mathrm{x}] \leq \mathrm{arr}[0]) \wedge \forall \mathrm{y}\, (\mathrm{j} < \mathrm{y} < \mathrm{len}(\mathrm{arr}) \longrightarrow \mathrm{arr}[\mathrm{y}] > \mathrm{arr}[0]) \wedge \mathrm{i} \leq \mathrm{j} + 1$$

Note that before the first iteration this is satisfied because arr[0] ≤ arr[0] and there are no elements in the range j+1..len(arr)-1. The loop condition (b) guarantees that i ≤ j.

In each iteration one of two things happens:

- i is incremented when arr[i] ≤ arr[0] – this leaves i ≤ j+1, and all elements up to index i are ≤ arr[0]
- j is decremented when arr[i] > arr[0], but only after swapping the item at i with the item at j. This leaves i ≤ j+1, and all elements down to index j are > arr[0]

After termination, we know (by the invariant) that i≤j+1, and by the negation of the loop condition ($\neg b$), that i > j. This implies that i = j+1.

$$\begin{aligned} P \wedge \neg b = {} & \forall x(0 \leq x < i \longrightarrow arr[x] \leq arr[0]) \\ & \wedge \forall y(j < y < len(arr) \longrightarrow arr[y] > arr[0]) \\ & \wedge i = \mathrm{j} + 1 \end{aligned}$$

If we just replace i with j+1 in the quantified portions of the proposition, then we see that $P \wedge \neg b \longrightarrow \mathrm{Q}$ directly.