Big-step

1. NEG $\dfrac{\langle e, \sigma \rangle \Downarrow_b v}{\langle !e, \sigma \rangle \Downarrow_b !v}$ where $v \in \{true, false\}$

2. For-F

$$\frac{\langle v := a_0, \sigma \rangle \Downarrow \sigma' \qquad \langle v \le a_1, \sigma' \rangle \Downarrow false}{\langle \text{for } v \text{ in } a_0 \text{ to } a_1 \text{ do } S, \sigma \rangle \Downarrow \sigma'}$$

For-T

$$\frac{\langle V := a_0, \sigma \rangle \Downarrow \sigma' \qquad \langle V \le a_1, \sigma' \rangle \Downarrow true \qquad \langle S, \sigma' \rangle \Downarrow \sigma'' \qquad \langle \text{for } v \text{ in } v+1 \text{ to } a_1 \text{ do } S, \sigma'' \rangle \Downarrow \sigma'''}{\langle \text{for } v \text{ in } a_0 \text{ to } a_1 \text{ do } S, \sigma \rangle \Downarrow \sigma'''}$$

3.

$$\text{Seq}\ \frac{\text{Assign}\ \dfrac{\text{Arith}\ \dfrac{\text{VAR}\ \dfrac{}{\langle a, \sigma \rangle \Downarrow 5} \qquad \text{VAR}\ \dfrac{}{\langle b, \sigma \rangle \Downarrow 10}}{\langle a+b, \sigma \rangle \Downarrow 15}}{\langle t := a+b, \sigma \rangle \Downarrow \sigma'} \qquad \text{Seq}\ \dfrac{\text{Assign}\ \dfrac{\text{VAR}\ \dfrac{}{\langle b, \sigma' \rangle \Downarrow 10}}{\langle a := b, \sigma' \rangle \Downarrow \sigma''} \qquad \text{Assign}\ \dfrac{\text{VAR}\ \dfrac{}{\langle t, \sigma'' \rangle \Downarrow 15}}{\langle b := t, \sigma'' \rangle \Downarrow \sigma'''}}{\langle a := b;\ b := t, \sigma' \rangle \Downarrow \sigma'''}}{\langle t := a+b;\ a := b;\ b := t, \sigma \rangle \Downarrow \sigma'''}$$

where

$\sigma = \{a := 5, b := 10\}$

$\sigma' = \{a := 5, b := 10, t := 15\}$

$\sigma'' = \{a := 10, b := 10, t := 15\}$

$\sigma''' = \{a := 10, b := 15, t := 15\}$

4. $$\dfrac{\dfrac{\text{VAR}\ \overline{\langle x,\sigma\rangle \Downarrow 10} \quad \text{VAR}\ \overline{\langle y,\sigma\rangle \Downarrow 20}}{\langle x > y,\sigma\rangle \Downarrow \text{false}}\ \text{REL} \qquad \dfrac{\dfrac{\text{VAR}\ \overline{\langle y,\sigma\rangle \Downarrow 20} \quad \text{LITERAL}\ \overline{\langle 10,\sigma\rangle \Downarrow 10}}{\langle y * 10,\sigma\rangle \Downarrow 200}\ \text{ARITH}}{\langle m := y*10,\ \sigma\rangle \Downarrow \sigma'}\ \text{ASSIGN}}{\langle \text{if } x>y \text{ then } m:=x*10 \text{ else } m:=y*10,\ \sigma\rangle \Downarrow \sigma'}\ \text{IF-F}$$

where $\sigma = \{x:=10,\ y:=20\}$

$\sigma' = \{x:=10,\ y:=20,\ m:=200\}$

5a. $\sigma' = \{ x := n+1,\ sum := \sum_{x=m}^{n} x \}$ // skipped false case

5b. $Q =$ "x:=m; while x≤n do sum:=sum+x; x:=x+1 od;"

5C. We need to proof P and Q generate the same environment after execution, we use ~~there~~ true and false cases to proof them identical.

(1) False case: $\sigma = \{x:=0, sum:=0\}$ $\sigma' = \{x:=m, sum:=0\}$.

P of For-F's proof tree:

$$\dfrac{\langle x := m, \sigma\rangle \Downarrow \sigma' \quad \langle x \le n, \sigma'\rangle \Downarrow \text{false}}{\langle \text{for } x \text{ in } m \text{ to } n \text{ do } sum:=sum+x, \sigma\rangle \Downarrow \sigma'}$$

Q of while-F's proof tree

$$\dfrac{\text{Assign}\ \overline{\langle x:=m, \sigma\rangle \Downarrow \sigma'} \qquad \text{While-F}\ \dfrac{\langle x \le n, \sigma'\rangle \Downarrow \text{false}}{\langle \text{while } x \le n \text{ do } sum:=sum+x; x:=x+1, \sigma'\rangle \Downarrow \sigma'}}{\langle x:=m; \text{ while } x\le n \text{ do } sum:=sum+x; x:=x+1 \text{ od}; \sigma\rangle \Downarrow \sigma'}\ \text{SEQ}$$

we note P and Q have same rules and generate the same σ' after execution, which means P and Q are equivalent in false case

(2) True case:

$$\sigma = \{x := 0,\ sum := 0\}$$
$$\sigma' = \{x := m,\ sum := 0\}$$
$$\sigma'' = \{x := m,\ sum := m\}$$
$$\sigma''' = \{x := m+1,\ sum := m\}$$
$$\sigma'''' = \{x := n+1,\ sum := \sum_{x=m}^{n} x\}.$$

P of For-T's proof tree.

$$\dfrac{\langle x := m, \sigma\rangle \Downarrow \sigma' \quad \langle x \le n, \sigma'\rangle \Downarrow true \quad \langle sum := sum + x, \sigma'\rangle \Downarrow \sigma'' \quad \dfrac{\langle x := x+1, \sigma''\rangle \Downarrow \sigma'''}{\langle \text{for } x \text{ in } x+1 \text{ to } n \text{ do } sum := sum + x, \sigma''\rangle \Downarrow \sigma''''}\ \text{For iteration}}{\langle \text{for } x \text{ in } m \text{ to } n \text{ do } sum := sum + x, \sigma\rangle \Downarrow \sigma''''}$$

Q of while-T's proof tree

$$\dfrac{\text{Assign}\ \langle x := m, \sigma\rangle \Downarrow \sigma' \quad \dfrac{\langle x \le n, \sigma'\rangle \Downarrow true \quad \text{Seq}\ \dfrac{\langle sum := sum + x, \sigma'\rangle \Downarrow \sigma'' \quad \text{Assign}\ \dfrac{}{\langle x := x+1, \sigma''\rangle \Downarrow \sigma'''}\ \text{precondition}}{\langle sum := sum + x;\ x := x+1, \sigma'\rangle \Downarrow \sigma'''}\ \text{while iteration}}{\langle \text{while } x \le n \text{ do } sum := sum + x;\ x := x+1, \sigma'\rangle \Downarrow \sigma''''}\ \text{while T}}{\langle x := m;\ \text{while } x \le n \text{ do } sum := sum + x;\ x := x+1 \text{ od}, \sigma\rangle \Downarrow \sigma''''}\ \text{Seq}$$

In the P's proof tree, the For-iteration has the same rules as Q's precondition combine while iteration. which generates the same environment $\sigma'''' = \{x := n+1,\ sum := \sum_{x=m}^{n} x\}$.

Therefore, P and Q are equivalent.