Sets, Relations, and Functions

CS 330: Discrete Structures
sets are unordered collections of objects
- if a set is finite, we can represent it by listing all its elements in curly braces:

\[ A = \{42, 9, 22, 78\} \]

- we can also specify a set by establishing a pattern and using ellipses:

\[ B = \{1, 3, 5, 7, \ldots\} \]

- but this can be ambiguous, so we prefer set builder notation:

\[ C = \{x \mid x \text{ is a prime number}\} \]
we use '∈' to express set membership.

Michael ∈ \{ x | x is faculty at IIT \}

W ∉ \{ c | c is a letter in the English alphabet \}

we can also use this in set builder notation:

C = \{ i | i is a positive integer \}

D = \{ j ∈ C | 100 ≤ j ≤ 200 \}

given predicate P(x), we define its truth set over domain D:

T_p = \{ x ∈ D | P(x) \}
∅ or ∅ denote the empty set.

Important: ∅ is not the empty set!

(it is a set containing one element, which happens to be the empty set — also: { ∅ })
\begin{align*}
T/F \quad ? \\
 a \in \{a,b,c\} & \quad T \quad \{a\} \not\in \{a,b,c\} & \quad F \\
 a \in \emptyset & \quad F \\
 \emptyset \in \{\emptyset\} & \quad F \\
 \emptyset = \{\} & \quad T \\
 2 \in \{w \mid 6 \not\leq \exists x \mid x \text{ is divisible by } w\} & \quad F
\end{align*}
Some fixed names we use for sets of numbers:

- \( \mathbb{N} \): natural numbers \( \{0, 1, 2, 3, \ldots \} \)
- \( \mathbb{Z} \): integers \( \{\ldots, -2, -1, 0, 1, 2, \ldots \} \)
- \( \mathbb{R} \): real numbers
- \( \mathbb{Q} \): rational numbers \( = \{ \frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0 \} \)

\( \mathbb{Z}^+ / \mathbb{Z}^- / \mathbb{R}^+ / \mathbb{R}^- / \mathbb{Q}^+ / \mathbb{Q}^- \) are the sets of positive/negative integers, reals, rationals.

E.g., \( \{x \in \mathbb{R} \mid x^2 < 9 \} \)
we can restrict the domain of a quantifier using sets:

e.g., universal quantification \( \forall x \, P(x) \) for domain \( D \):

\[
\forall x \in D \, (P(x))
\equiv \forall x \, (x \in D \implies P(x))
\]

e.g. existential quantification \( \exists x \, P(x) \) for domain \( D \):

\[
\exists x \in D \, (P(x))
\equiv \exists x \, (x \in D \land P(x))
\]
Sets are equal if they contain the same distinct elements. (order doesn't matter, duplicates don't matter!)

equal or not?

\[ \{1, 2, 3, 4\} = \{4, 3, 2, 1\} \]

\[ \{1, 1, 2, 3, 2, 2, 4\} = \{1, 2, 3, 4\} \]

\[ \{x \in \mathbb{N} \mid x < 4\} = \{x \in \mathbb{Z} \mid 0 \leq x < 4\} \]

\[ \{x \in \mathbb{Z} \mid x^2 < x\} = \{x \in \mathbb{R} \mid x^2 < 0\} \]
the **cardinality** of a finite set $A$, denoted $|A|$, is the number of distinct elements in $A$.

| $\{0, 1, 2, 3\}$ | $= 4$ 
|-------------------|--------|
| $\emptyset$       | $= 0$  
| $\{2, 2, 3, 1, 2, 3\}$ | $= 3$  
| $\{x \mid x \text{ is prime and } x < 10\}$ | $= 4$  

a finite set is countable.

an infinite set is countable if we can find a one-to-one correspondence between its elements and the natural numbers.

(CS intuition: countable if we can “index” its elements)
is the set of even numbers \( \{0, 2, 4, \ldots \} \) countable?

\[ N = 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ldots \]

\[ \downarrow \downarrow \downarrow \downarrow \downarrow \quad \text{mapping function: } f(n) = 2n \]

\[ \text{evens} = 0 \ 2 \ 4 \ 6 \ 8 \ 10 \ 12 \ldots \]

there is a one-to-one correspondence between \( N \) and evens.

Ans. Yes!
is \( \mathbb{Z} \) (integers \{ \ldots -3, -2, -1, 0, 1, 2, 3, \ldots \}) countable?

\[
\mathbb{N} = \{0, 1, 2, 3, 4, 5, 6, \ldots \}
\]

rewire \( \mathbb{Z} \) as \( \{0, 1, -1, 2, -2, 3, -3, \ldots \} \)

Ans. Yes!
Is \( \mathbb{Q} = \left\{ \frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0 \right\} \) countable?

\[ \begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & \ldots \\
0 & 1/2 & 2/3 & 3/4 & 4/5 & \ldots \\
1/2 & 2/3 & 3/4 & 4/5 & \ldots \\
3/2 & 6/5 & \ldots \\
4/3 & \ldots \\
5/4 & \ldots \\
\end{array} \]

Answer: Yes!
is $\mathbb{R}$ (real numbers) countable?

- Let's assume yes

- Consider all reals between 0 and 1:

  $0 \rightarrow 0. a_{00} a_{01} a_{02} a_{03}$
  $1 \rightarrow 0. a_{10} a_{11} a_{12} a_{13}$
  $2 \rightarrow 0. a_{20} a_{21} a_{22} a_{23}$
  $3 \rightarrow 0. a_{30} a_{31} a_{32} a_{33}$
  ...
  $n \rightarrow 1.00000\ldots$

Let's pick digits on diagonal and make a new real where its digits differ from these.

E.g., $b_{ij} = (a_{ij} + 1) \mod 10$

This new real is not in our list!

$\therefore \mathbb{R}$ is NOT countable.
set operations, given sets $A$ and $B$:

- **union**: $A \cup B$
  \[ A \cup B = \{ x \mid x \in A \lor x \in B \} \]

- **intersection**: $A \cap B$
  \[ A \cap B = \{ x \mid x \in A \land x \in B \} \]

- **difference**: $A - B$ or $A \setminus B$
  \[ A - B = \{ x \mid x \in A \land x \notin B \} \]

- **complement**: $\overline{A}$
  \[ \overline{A} = \{ x \in U \mid x \notin A \} \]
  "universal" set

**Venn Diagrams**

- $A \cup B$
- $A \cap B$
- $A - B$
- $\overline{A}$
### Table 1: Set Identities.

<table>
<thead>
<tr>
<th>Identity</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \cap U = A )</td>
<td>Identity laws</td>
</tr>
<tr>
<td>( A \cup \emptyset = A )</td>
<td></td>
</tr>
<tr>
<td>( A \cup U = U )</td>
<td>Domination laws</td>
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<tr>
<td>( A \cap \emptyset = \emptyset )</td>
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</tr>
<tr>
<td>( A \cup A = A )</td>
<td>Idempotent laws</td>
</tr>
<tr>
<td>( A \cap A = A )</td>
<td></td>
</tr>
<tr>
<td>( \overline{A} = A )</td>
<td>Complementation law</td>
</tr>
<tr>
<td>( A \cup B = B \cup A )</td>
<td>Commutative laws</td>
</tr>
<tr>
<td>( A \cap B = B \cap A )</td>
<td></td>
</tr>
<tr>
<td>( A \cup (B \cup C) = (A \cup B) \cup C )</td>
<td>Associative laws</td>
</tr>
<tr>
<td>( A \cap (B \cap C) = (A \cap B) \cap C )</td>
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<tr>
<td>( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) )</td>
<td>Distributive laws</td>
</tr>
<tr>
<td>( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) )</td>
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</tr>
<tr>
<td>( \overline{A \cap B} = \overline{A} \cup \overline{B} )</td>
<td>De Morgan’s laws</td>
</tr>
<tr>
<td>( \overline{A \cup B} = \overline{A} \cap \overline{B} )</td>
<td></td>
</tr>
<tr>
<td>( A \cup (A \cap B) = A )</td>
<td>Absorption laws</td>
</tr>
<tr>
<td>( A \cap (A \cup B) = A )</td>
<td></td>
</tr>
<tr>
<td>( A \cup \overline{A} = U )</td>
<td>Complementation laws</td>
</tr>
<tr>
<td>( A \cap \overline{A} = \emptyset )</td>
<td></td>
</tr>
</tbody>
</table>

**Proof:**

\[ \overline{A \cup B} = \overline{A} \cap \overline{B} : \]

\[ \overline{A \cup B} = \{ x \mid x \notin (A \cup B) \} \]
\[ = \{ x \mid \neg (x \in (A \cup B)) \} \]
\[ = \{ x \mid \neg (x \in A \lor x \in B) \} \]
\[ = \{ x \mid x \notin A \land x \notin B \} \]
\[ = \{ x \mid x \in \overline{A} \land x \in \overline{B} \} \]
\[ = \overline{A} \land \overline{B} \]
\[ A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_k = \bigcup_{i=1}^{k} A_i \]

\[ A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_k = \bigcap_{i=1}^{k} A_i \]
we say \( A \) is a subset of \( B \) \( (A \subseteq B) \), and \( B \) is a superset of \( A \) \( (B \supseteq A) \), iff every element in \( A \) is also in \( B \). i.e.,

\[
\forall x (x \in A \rightarrow x \in B)
\]

we say \( A \) is a proper subset of \( B \) \( (A \subset B) \) iff \( A \) is a subset of \( B \) but \( A \neq B \). i.e.,

\[
\forall x (x \in A \rightarrow x \in B) \land \exists x (x \in B \land x \notin A)
\]
the power set of $S$, $\mathcal{P}(S)$, is the set of all subsets of $S$

$$\mathcal{P}(\{a, b, c\}) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

$$\mathcal{P}(\{a\}) = \{ \emptyset, \{a\}\}$$

$$\mathcal{P}(\emptyset) = \{ \emptyset \}$$

For set $S$ with $|S| = n$, $|\mathcal{P}(S)| = 2^n$
sets are unordered, but sometimes order matters!

an \textit{n-tuple} is an ordered collection of \textit{n} elements

\[(1, 2), (2, 1), (1, 1, 2), (2, 2, 1, 2, 1)\] are all distinct!

\textit{“ordered pairs”}
the **Cartesian product** of two sets $A$ and $B$ is:

$$A \times B = \{(a,b) \mid a \in A \land b \in B\}$$

**e.g.** $\{1,2,3\} \times \{p,q\} = \{(1,p),(1,q), (2,p),(2,q), (3,p),(3,q)\}$

$\{x,y,z\} \times \emptyset = \emptyset$

$\{x,y,z\} \times \{\emptyset\} = \{(x,\emptyset),(y,\emptyset),(z,\emptyset)\}$
the Cartesian Product of sets $A_1, A_2, \ldots, A_n$ is defined as:

$$A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, \ldots, n\}$$

e.g. $\{a, b\} \times \{c, d\} \times \{e, f\} = \{(a, c, e), (a, c, f), (a, d, e), (a, d, f), (b, c, e), (b, c, f), (b, d, e), (b, d, f)\}$

we also write $A^2$ for $A \times A$, $A^3$ for $A \times A \times A$, i.e.,

$$A^n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A \text{ for } i = 1, 2, \ldots, n\}$$

e.g. $\{a, b, c\}^2 = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$
Note: \( A \times B \neq B \times A \) (unless \( A \) or \( B \) is \( \emptyset \)), and \( A \times B \times C \neq (A \times B) \times C \)!

e.g. \( \left( \{a, b\} \times \{c, d\} \right) \times \{e, f\} \)

\[
= \{(a, c), (a, d), (b, c), (b, d)\} \times \{e, f\}\\
= \{(a, c, e), (a, c, f), (a, d, e), (a, d, f), (b, c, e), (b, c, f), (b, d, e), (b, d, f)\}
\]
A relation from set $A$ to set $B$ is a subset of $A \times B$.

E.g., given $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$, write down a relation from $A$ to $B$:

$$G = \{(1, b), (3, c), (2, a), (2, c)\}$$

- 3 is related to c through $G$: $3 R c$
- 3 is not related to $b$ through $G$: $3 \not R b$

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>X</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A relation on a set $A$ is a relation from $A$ to $A$ (i.e., a subset of $A \times A$)

e.g., given $A = \{1, 2, 3, 4 \}$, write down a relation on $A$

$H = \{(1,2), (1,3), (2,4), (4,1)\}$
Matrix representation

We can represent a relation from \( A = \{a_1, a_2, ..., a_m \} \) to \( B = \{b_1, b_2, ..., b_n \} \) as the matrix \( M_R = [m_{ij}] \), where

\[
m_{ij} = \begin{cases} 
1 & \text{if } (a_i, b_j) \in R \\
0 & \text{if } (a_i, b_j) \notin R 
\end{cases}
\]

e.g., given \( A = \{a, b, c\} \), \( B = \{1, 2\} \) and \( R = \{(a, 1), (a, 2), (b, 2), (c, 1)\} \), draw the matrix for \( R \).
We can represent a relation on a set as a directed graph (digraph), where vertices correspond to elements and edges are drawn between vertices for all ordered pairs in the relation.

E.g., given $A = \{1, 2, 3, 4\}$ and relation $R = \{(1,1), (1,2), (2,3), (2,4), (3,2), (3,3), (4,1), (4,2), (4,4)\}$, draw the digraph for $R$. 

\[ \text{Diagram:} \]

- Vertices: 1, 2, 3, 4
- Edges: 
  - 1 → 1
  - 1 → 2
  - 2 → 3
  - 2 → 4
  - 3 → 2
  - 3 → 3
  - 4 → 1
  - 4 → 2
  - 4 → 3
  - 4 → 4
Relation $R$ on a set $A$ is ... (where $a, b, c \in A$)

- **reflexive** if $\forall a ((a,a) \in R)$
- **symmetric** if $\forall a \forall b ((a,b) \in R \rightarrow (b,a) \in R)$
- **antisymmetric** if $\forall a \forall b ((a,b) \in R \land (b,a) \in R) \rightarrow (a=b)$
- **transitive** if $\forall a \forall b \forall c ((a,b) \in R \land (b,c) \in R) \rightarrow (a,c) \in R$
What do matrix and digraph representations of reflexive, symmetric, antisymmetric, and transitive relations look like?

**Reflexive:**

$$\begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{bmatrix}$$

**Symmetric:**

$$\begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{bmatrix}$$

**Antisymmetric:**

$$\begin{bmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}$$

**Transitive:**

$$\begin{bmatrix}
? & ? & ? \\
? & ? & ? \\
? & ? & ? \\
\end{bmatrix}$$

(examine $M^2$)
A relation on a set is an equivalence relation if it is simultaneously reflexive, symmetric, and transitive.

Two elements $a$ and $b$ related by an equivalence relation are called equivalent ($a \sim b$).
e.g., consider the relation:

\[ R = \{(a, b) \mid a, b \text{ are strings of English letters, and } \]
\[ a \mathbin{R} b \iff \text{length}(a) = \text{length}(b) \} \]

is \( R \) an equivalence relation? Yes!

- **Reflexivity**: \( \text{length}(a) = \text{length}(a) \) \( \checkmark \)
- **Symmetry**: \( (\text{len}(a) = \text{len}(b)) \Rightarrow (\text{len}(b) = \text{len}(a)) \) \( \checkmark \)
- **Transitivity**: \( ((\text{len}(a) = \text{len}(b)) \land (\text{len}(b) = \text{len}(c))) \Rightarrow (\text{len}(a) = \text{len}(c)) \) \( \checkmark \)
congruence mod N: \( a \equiv b \pmod{N} \), which is true iff \( a-b \) is evenly divisible by \( N \).

\[ R = \{(a,b) \mid a \mod N = b \mod N \} \text{ for } a, b \in \mathbb{Z} \]

is \( R \) an equivalence relation? Yes!

- reflexivity: \( a \equiv a \pmod{N} \), \( 0 = kN \checkmark \)
- symmetry: \( a \equiv b \pmod{N} \rightarrow a-b = kN \)
  \[ b-a = -kN \therefore b \equiv a \mod N \checkmark \]
- transitivity: \( a \equiv b \pmod{N} \) and \( b \equiv c \pmod{N} \)
  \[ \rightarrow a-b = kN \text{ and } b-c = lN \]
  \[ a-b + b-c = kN + lN, a-c = (k+l)N \]
  \[ \therefore a \equiv c \pmod{N} \checkmark \]
If $R$ is an equivalence relation on set $A$, the **equivalence class** of $a \in A$, denoted $[a]_R$, is the set $\{e \mid (a, e), (e, a) \in R\}$. 

For $b (b \in [a]_R)$, $b$ is a **representative** of $[a]_R$.

e.g., what are the equivalence classes of 0, 1, 2 for the relation $R = \{(a, b) \mid a \equiv b \pmod{3}\}$, $a, b \in \mathbb{Z}$? 

$[0]_R = \{\ldots, -9, -6, -3, 0, 3, 6, 9, \ldots \}$

$[1]_R = \{\ldots, -8, -5, -2, 1, 4, 7, 10, \ldots \}$

$[2]_R = \{\ldots, -7, -4, -1, 2, 5, 8, 11, \ldots \}$
An equivalence relation $R$ on set $A$ \textcolor{yellow}{partitions} set $A$ i.e., the equivalence classes of distinct elements $a, b$ are either equal or \textcolor{yellow}{disjoint}:

\[
[a]_R \neq [b]_R \iff [a]_R \cap [b]_R = \emptyset
\]

the union of all equivalence classes of $R$ is $A$:

\[
\bigcup_{a \in A} [a]_R = A
\]
A function $f$ from set $A$ to set $B$ is a relation from $A$ to $B$ that assigns exactly one element of $B$ to each element of $A$.

- we write $f : A \to B$ (f maps set A to set B) and $f(a) = b$ (f assigns $b \in B$ to $a \in A$)

- $A$ is the domain of $f$, and $B$ is the codomain

- The set of all elements of $B$ assigned by $f$ is called the range
types of functions

one-to-one/injection
no element of codomain is assigned to more than one value in domain

onto/surjection
all elements of codomain are assigned

one-to-one correspondence/bijection
both one-to-one and onto

partial
not all values of domain have an assignment (i.e., they may be undefined)
e.g., functions from $A$ to $B$:

- injection & not surjection

- surjection & not injection

- bijection

- partial function

- not injection & not surjection

- not a function
If a function $f$ is a bijection, we can define its inverse, $f^{-1}$, where $f^{-1}(b) = a$ for every $f(a) = b$.

e.g. draw the inverse of

\[ f \]

\[
\begin{array}{ccc}
1 & \rightarrow & a \\
2 & \rightarrow & b \\
3 & \rightarrow & c \\
\end{array}
\]

\[ f^{-1} \]

\[
\begin{array}{ccc}
a & \rightarrow & 1 \\
b & \rightarrow & 2 \\
c & \rightarrow & 3 \\
\end{array}
\]

e.g., is the function $f(x) = 2x, x \in \mathbb{R}$ invertible? Yes

e.g., is the function $f(x) = x^2 + 1, x \in \mathbb{R}$ invertible? No
The composition of \( f : A \to B \) and \( g : B \to C \), is defined as the function \((g \circ f) : A \to C\) where, for all \( a \in A \),

\[
(g \circ f)(a) = g(f(a))
\]

e.g., given \( A = \{a, b, c\} \), \( B = \{1, 2, 3\} \), \( C = \{h, i, j\} \), \( f = \{(a, 1), (b, 3), (c, 3)\} \), \( g = \{(1, h), (2, i), (3, j)\} \),

write down the function \( g \circ f \)

\[
g \circ f = \{(a, h), (b, j), (c, j)\}
\]
e.g., given functions \( f: \mathbb{R} \rightarrow \mathbb{R} \) and \( g: \mathbb{R} \rightarrow \mathbb{R} \), where
\[
f(x) = 2x + 1, \quad g(x) = 4x^2,
\]
find \( f \circ g \) and \( g \circ x \).

\[
(f \circ g)(x) = f(g(x)) = 2(4x^2) + 1 = 8x^2 + 1
\]
\[
(g \circ f)(x) = g(f(x)) = 4(2x + 1)^2 = 4(4x^2 + 4x + 1)
\]
\[
= 16x^2 + 16x + 4
\]
Relationships between more than 2 sets can be expressed using **n-ary relations** (aka. relations of degree n)

Given sets \( A_1, A_2, \ldots, A_n \), an n-ary relation on these sets is a subset of \( A_1 \times A_2 \times \ldots \times A_n \) (which, recall, is a set of n-tuples \( (a_1, a_2, \ldots, a_n) \) where \( a_i \in A_i \))

E.g., give a relation of degree 4 on the set \( \mathbb{N} \).

\[ R \subseteq \mathbb{N} \times \mathbb{N} \times \mathbb{N} \times \mathbb{N} \]

\[ R = \{ (0, 0, 0, 0), (0, 1, 3, 100), (2, 1, 4, 3) \} \]
e.g., give a relation $R$ on $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$, where each triple $(a, b, c) \in R$ iff $a < b + c$

$$R = \{(0, 1, 2), (100, 50, 60), (20, -20, 50)\}$$

e.g., describe an $n$-ary relation that can be used to represent airport flight information

- **A**: airport codes = $\{\text{ORD}, \text{MDW}, \text{JFK}, \text{LAX}, \ldots\}$
- **B**: airline codes = $\{\text{AA}, \text{DL}, \text{SW}, \ldots\}$
- **C**: flight numbers = $\mathbb{N}$
- **D**: time = $\{0:00, 0:01, \ldots, 23:59\}$
- **E**: remark = $\{\text{landed}, \text{departed}, \text{boarding}, \ldots\}$

relation $R \subseteq A \times B \times C \times B \times C \times D \times E$
Relational databases are conceptually built on top of storing all data as tuples grouped by relation, managed by operations defined using predicate logic.
A relational database organizes data into relations over groups of attributes, and supports operations defined using predicate logic to query and manage that data.

E.g., contact database:

<table>
<thead>
<tr>
<th>F</th>
<th>L</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>first name</td>
<td>last name</td>
<td>email</td>
</tr>
<tr>
<td>John</td>
<td>Doe</td>
<td><a href="mailto:jdoe@a.com">jdoe@a.com</a></td>
</tr>
<tr>
<td>Mary</td>
<td>Jane</td>
<td><a href="mailto:mjane@b.com">mjane@b.com</a></td>
</tr>
</tbody>
</table>

\[ R \subseteq F \times L \times E \]

\{ (John, Doe, jdoe@a.com), (Mary, Jane, mjane@b.com), \ldots \} 

“table” = relation
“attribute/column” = domain
“row/record” = tuple
A domain of a relation is called the primary key when the value from this domain is unique across all tuples.

e.g., | sid | name | email | phone |
<table>
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When the value from a combination of multiple domains is unique across tuples, these domains are collectively a composite key.

e.g., | dept | course # | title |
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<thead>
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<tbody>
<tr>
<td>CS</td>
<td>330</td>
<td>DisC. Stu.</td>
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<tr>
<td>CS</td>
<td>331</td>
<td>Data Stu.</td>
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</tbody>
</table>
Database operations:

- **selection**: $\sigma_C(R)$ — selects all tuples from $R$ for which the predicate $C$ is true.

- **projection**: $\Pi_{a_1, a_2, \ldots, a_n}(R)$ — evaluates to the tuples from $R$, up only values from the attributes $a_1, a_2, \ldots, a_n$ included.

- **join**: $\bowtie(R, S)$ — evaluates to combined tuples from $R$ and $S$, based on common attributes.
Database + SQL Demo