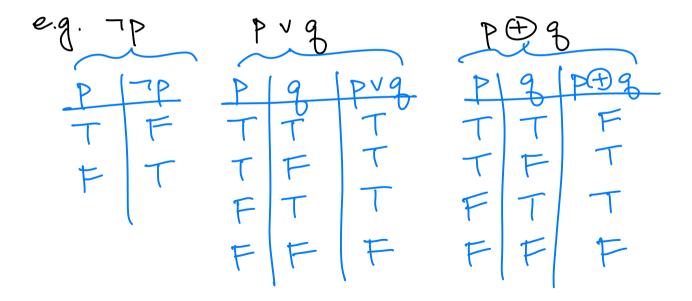
Propositional Logic CS 320 : Discrete Structures

"proposition": a declarative sentence that is enter  

$$True(T/1)$$
 or False  $(F/0/L)$   
"totom" ("  
"falsow" "  
"also of the sentence that is enter  
"totom" ("  
"falsow" ("  
"also of the sentence that is enter  
"totom" ("  
"falsow" ("  
"also of the sentence that is enter  
"totom" ("  
"totom" ("  
"also of the sentence that is enter  
"totom" ("  
"totom" ("

we use truth tables to show the value of a proposition for all contrinations of values taken by its variables.



how many nows	in a truth table for	a proposition of M	J variables?
e.g. N= 2	eg N=3	eg N = 4	$\frac{1}{2^N}$
$ \begin{array}{c} P \\ P \\ T \\ T \\ F \\ F$	$\begin{array}{c c} P & r \\ \hline P & T \\ \hline T & F \\ \hline T & F \\ \hline T & F \\ \hline T \\ T \\$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$2^{4} = 16$
		000	

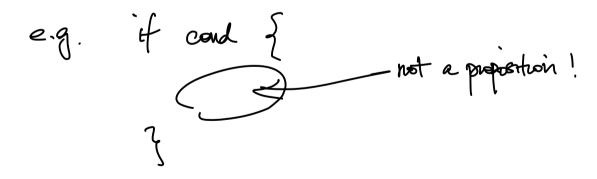
Applying logic to English propositions:  

$$P: "I have cats"$$
  
 $g: "computer science is a science"$   
 $r: "F < 100"$ 

read: p1g ¬p v ¬g p (g v ¬r) p ¬ P g v ¬g

 $p \rightarrow q$  is a proposition known as a conditional statement (aka. implication) read "if p, then q" hypothesis/ conclusion/ antecedent conceptent

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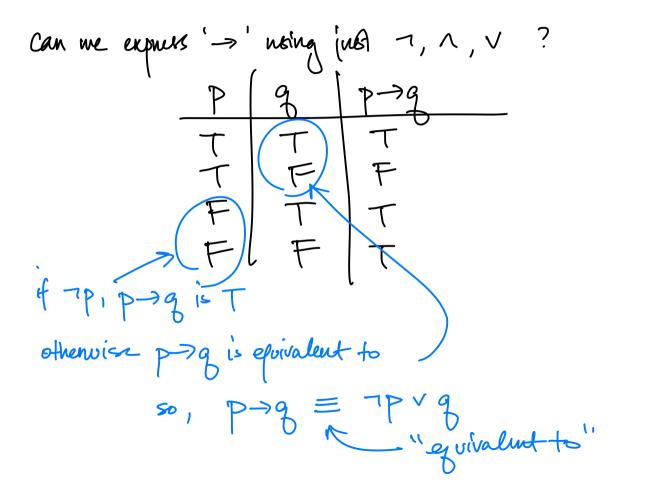
eg." if the Ac is an , then 1/11 be cold."  

$$p: the Ac is on
g: 1/11 be cold.
truthe table for  $p \rightarrow g$   

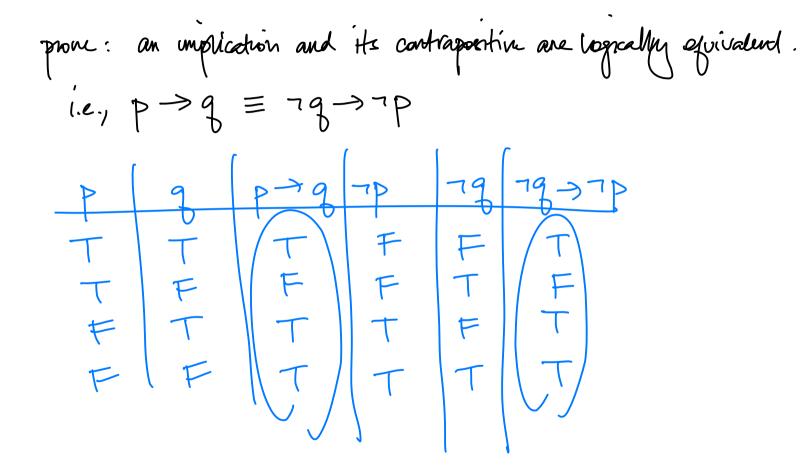
$$\frac{P(g)}{T} \frac{p \rightarrow g}{T}$$

$$\frac{P(g)}{T} \frac{p \rightarrow g}{T}$$

$$\frac{F}{T} \frac{T}{T} \frac{T}{T} \frac{T}{T} \cdots \frac{T}{T} \frac{T}{T$$$$



mvern:



$$P \leqslant q \text{ cupnesses the biconditional statement "pitand only if q", which is equivalent to the proposition  $(p > q) \land (q = p)$   
-truth table:  
$$\frac{p \quad q \quad p \leqslant q}{T \quad T \quad T \quad F}$$
$$\frac{T \quad T \quad F}{F \quad F \quad F}$$
$$F \quad T \quad F \quad F$$$$

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$$p \equiv q$$
 (p is equivalent to q) if  $p \ll q$  is a tantology.

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TABLE 6 Logical Equivalences.			
Equivalence	Name		
$p \land \mathbf{T} \equiv p$ $p \lor \mathbf{F} \equiv p$	Identity laws		
$p \lor \mathbf{T} \equiv \mathbf{T}$ $p \land \mathbf{F} \equiv \mathbf{F}$	Domination laws		
$p \lor p \equiv p$ $p \land p \equiv p$	Idempotent laws		
$\neg(\neg p) \equiv p$	Double negation law		
$p \lor q \equiv q \lor p$ $p \land q \equiv q \land p$	Commutative laws		
$(p \lor q) \lor r \equiv p \lor (q \lor r)$ $(p \land q) \land r \equiv p \land (q \land r)$	Associative laws		
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Distributive laws		
$\neg (p \land q) \equiv \neg p \lor \neg q$ $\neg (p \lor q) \equiv \neg p \land \neg q$	De Morgan's laws		
$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption laws		
$p \lor \neg p \equiv \mathbf{T}$ $p \land \neg p \equiv \mathbf{F}$	Negation laws		

TABLE 7 Logical Equivalences  
Involving Conditional  
Statements.
$$p \to q \equiv \neg p \lor q$$
 $p \to q \equiv \neg p \lor q$  $p \lor q \equiv \neg p \to q$  $p \lor q \equiv \neg p \to q$  $p \lor q \equiv \neg p \to q$  $p \land q \equiv \neg (p \to \neg q)$  $\neg (p \to q) \equiv p \land \neg q$  $(p \to q) \land (p \to r) \equiv p \to (q \land r)$  $(p \to r) \land (q \to r) \equiv (p \lor q) \to r$  $(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$  $(p \to r) \lor (q \to r) \equiv (p \land q) \to r$ 

TABLE 8 Logical<br/>Equivalences Involving<br/>Biconditional Statements. $p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$  $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$  $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$  $\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$ 

Note that the propositional variables listed in the preceding  
tables can stand for compound propositions.  
E.g. De Morgan's Law: 
$$\neg (p \land q) \equiv \neg p \lor \neg q$$
  
 $\neg ((r \land \neg s) \land (\neg t \Rightarrow u)) \equiv \neg (r \land \neg s) \lor \neg (\neg t \Rightarrow u)$