Exercises from Section 7.1: (1), (3), (9), (11), (25)

1. What probability should be assigned to the outcome of heads when a biased coin is tossed, if heads is three times as likely to come up as tails? What probability should be assigned to the outcome of tails?

3. Find the probability of each outcome when a biased die is rolled, if rolling a 2 or rolling a 4 is three times as likely as rolling each of the other four numbers on the die and it is equally likely to roll a 2 or a 4.

9. What is the probability of these events when we randomly select a permutation of the 26 lowercase letters of the English alphabet?

- **a)** The permutation consists of the letters in reverse alphabetic order.
- **b)** z is the first letter of the permutation.
- **c)** z precedes a in the permutation.
- **d)** a immediately precedes z in the permutation.
- **e)** a immediately precedes m, which immediately precedes z in the permutation.
- **f)** m, n, and o are in their original places in the permutation.

11. Suppose that E and F are events such that $p(E) = 0.7$ and $p(F) = 0.5$. Show that $p(E \cup F) \geq 0.7$ and $p(E \cap F) \geq 0.2$.

25. What is the conditional probability that a randomly generated bit string of length four contains at least two consecutive 0s, given that the first bit is a 1? (Assume the probabilities of a 0 and a 1 are the same.)

Exercises from Section 7.2: (3), (5), (7), (19)

3. Find the probability of each outcome when a biased die is rolled, if rolling a 2 or rolling a 4 is three times as likely as rolling each of the other four numbers on the die and it is equally likely to roll a 2 or a 4.

5. A pair of dice is loaded. The probability that a 4 appears on the first die is $2/7$, and the probability that a 3 appears on the second die is $2/7$. Other outcomes for each die appear with probability $1/7$. What is the probability of 7 appearing as the sum of the numbers when the two dice are rolled?

7. What is the probability of these events when we randomly select a permutation of $\{1, 2, 3, 4\}$?

- **a)** 1 precedes 4.
- **b)** 4 precedes 1.
- **c)** 4 precedes 1 and 4 precedes 2.
- **d)** 4 precedes 1, 4 precedes 2, and 4 precedes 3.
- **e)** 4 precedes 3 and 2 precedes 1.

19.
- **a)** What is the probability that two people chosen at random were born during the same month of the year?
- **b)** What is the probability that in a group of n people chosen at random, there are at least two born in the same month of the year?
- **c)** How many people chosen at random are needed to make the probability greater than $1/2$ that there are at least two people born in the same month of the year?

Exercises from Section 7.3: (5), (7)

5. Suppose that 8% of all bicycle racers use steroids, that a bicyclist who uses steroids tests positive for steroids 96% of the time, and that a bicyclist who does not use steroids tests positive for steroids 9% of the time. What is the probability that a randomly selected bicyclist who tests positive for steroids actually uses steroids?

7. Suppose that a test for opium use has a 2% false positive rate and a 5% false negative rate. That is, 2% of people who do not use opium test positive for opium, and 5% of opium users test negative for opium. Furthermore, suppose that 1% of people actually use opium.

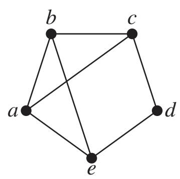
a) Find the probability that someone who tests negative for opium use does not use opium.

b) Find the probability that someone who tests positive for opium use actually uses opium.

Exercises from Section 10.5: (1), (3), (5), (31), (33), (35), (47)

In Exercises 1–8 determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

1.



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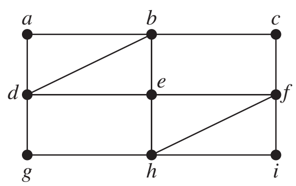

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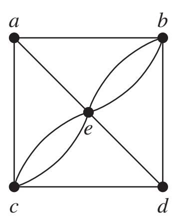

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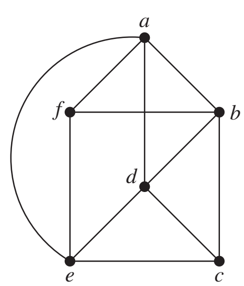

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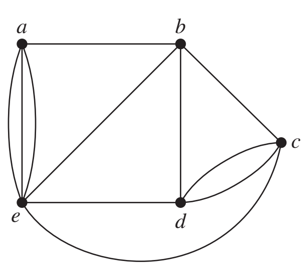
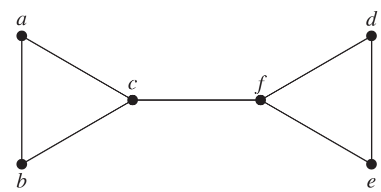
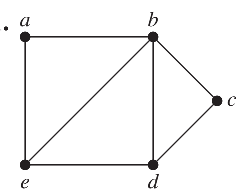


In Exercises 30–36 determine whether the given graph has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.

30.



31.



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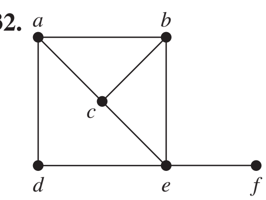

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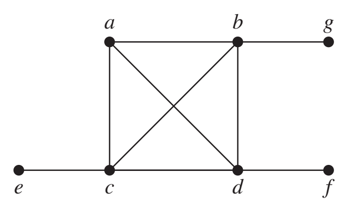
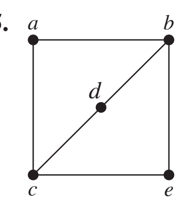
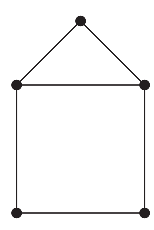


35.

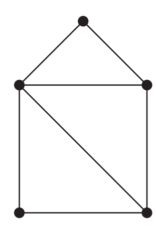


47. For each of these graphs, determine (*i*) whether Dirac's theorem can be used to show that the graph has a Hamilton circuit, (*ii*) whether Ore's theorem can be used to show that the graph has a Hamilton circuit, and (*iii*) whether the graph has a Hamilton circuit.

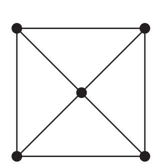
a)



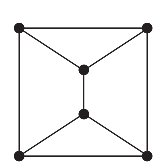
b)



c)



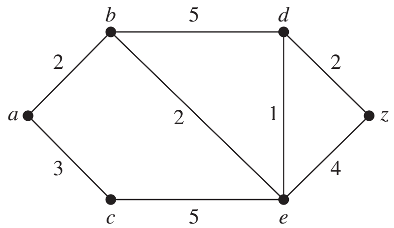
d)



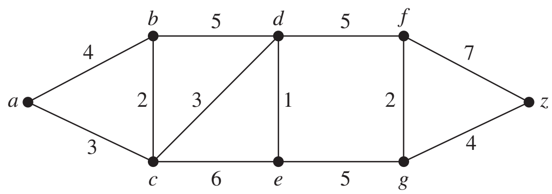
Exercises from Section 10.6: (3), (17), (25)

In Exercises 2–4 find the length of a shortest path between a and z in the given weighted graph.

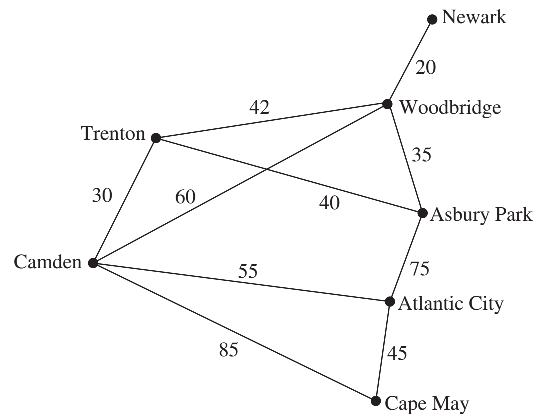
2.



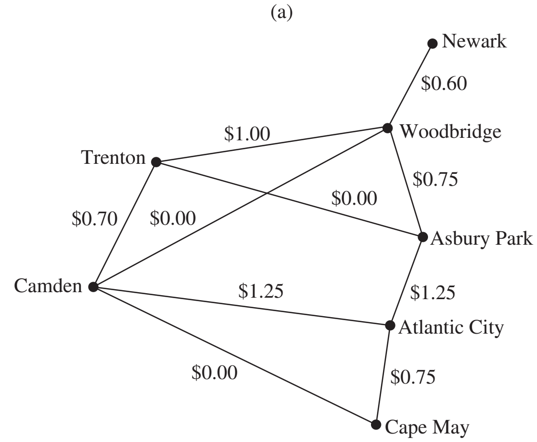
3.



17. The weighted graphs in the figures here show some major roads in New Jersey. Part (a) shows the distances between cities on these roads; part (b) shows the tolls.



(a)

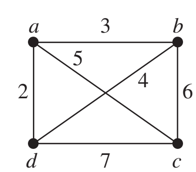


(b)

a) Find a shortest route in distance between Newark and Camden, and between Newark and Cape May, using these roads.

b) Find a least expensive route in terms of total tolls using the roads in the graph between the pairs of cities in part (a) of this exercise.

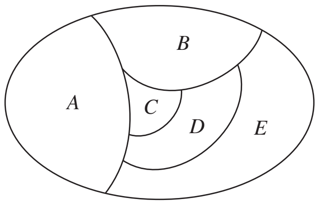
25. Solve the traveling salesperson problem for this graph by finding the total weight of all Hamilton circuits and determining a circuit with minimum total weight.



Exercises from Section 10.8: (3), (9), (11)

In Exercises 1–4 construct the dual graph for the map shown. Then find the number of colors needed to color the map so that no two adjacent regions have the same color.

1.



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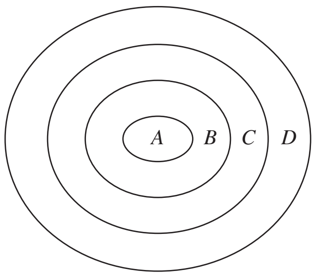
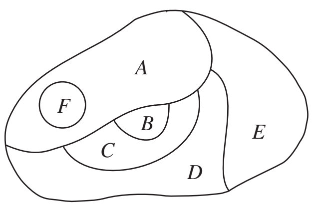
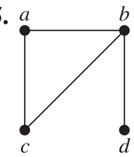


3.



In Exercises 5–11 find the chromatic number of the given graph.

5.



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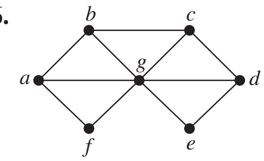

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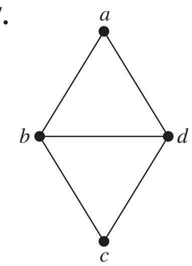
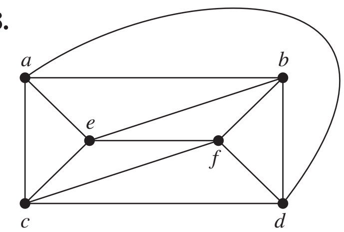
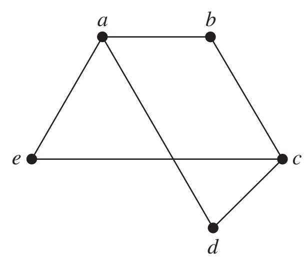


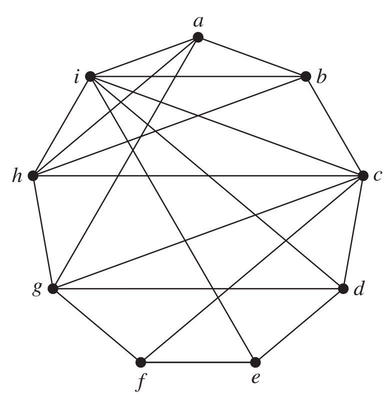
8.



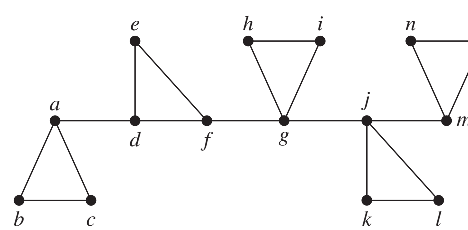
9.



10.



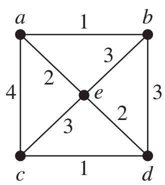
11.



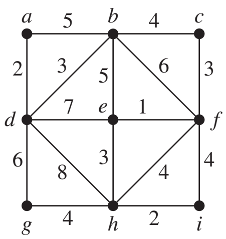
Exercises from Section 11.5: (3), (7), (11)

In Exercises 2–4 use Prim's algorithm to find a minimum spanning tree for the given weighted graph.

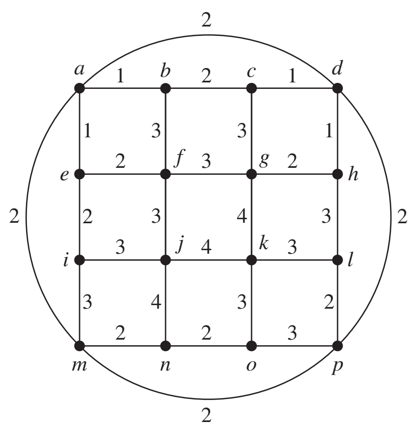
2.



3.



4.



7. Use Kruskal's algorithm to find a minimum spanning tree for the weighted graph in Exercise 3.

A **maximum spanning tree** of a connected weighted undirected graph is a spanning tree with the largest possible weight.

11. Devise an algorithm similar to Prim's algorithm for constructing a maximum spanning tree of a connected weighted graph.