Exercises from Section 6.1: (14), (32), (42), (46), (50), (58)

14. How many bit strings of length n, where n is a positive integer, start and end with 1s?

32. How many strings of eight uppercase English letters are there

a) if letters can be repeated?
b) if no letter can be repeated?
c) that start with X, if letters can be repeated?
d) that start with X, if no letter can be repeated?
e) that start and end with X, if letters can be repeated?
f) that start with the letters BO (in that order), if letters can be repeated?
g) that start and end with the letters BO (in that order), if letters can be repeated?
h) that start or end with the letters BO (in that order), if letters can be repeated?

42. How many 4-element DNA sequences

a) do not contain the base T?
b) contain the sequence ACG?
c) contain all four bases A, T, C, and G?
d) contain exactly three of the four bases A, T, C, and G?

46. How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?

50. How many bit strings of length seven either begin with two 0s or end with three 1s?

58. The name of a variable in the C programming language is a string that can contain uppercase letters, lowercase letters, digits, or underscores. Further, the first character in the string must be a letter, either uppercase or lowercase, or an underscore. If the name of a variable is determined by its first eight characters, how many different variables can be named in C? (Note that the name of a variable may contain fewer than eight characters.)

Exercises from Section 6.2: (6), (10), (26* — not graded, but attempt for a challenge!), (28)

6. There are six professors teaching the introductory discrete mathematics class at a university. The same final exam is given by all six professors. If the lowest possible score on the final is 0 and the highest possible score is 100, how many students must there be to guarantee

10. Show that if f is a function from S to T, where S and T are finite sets with $|S| > |T|$, then there are elements s_1 and s_2 in S such that $f(s_1) = f(s_2)$, or in other words, f is not one-to-one.

26. Suppose that 21 girls and 21 boys enter a mathematics competition. Furthermore, suppose that each entrant solves at most six questions, and for every boy-girl pair, there is at least one question that they both solved. Show that there is a question that was solved by at least three girls and at least three boys.

28. Show that in a group of five people (where any two people are either friends or enemies), there are not necessarily three mutual friends or three mutual enemies.

Exercises from Section 6.3: (12), (18), (40), (44), (46)

12. How many bit strings of length 12 contain

a) exactly three 1s?
b) at most three 1s?
c) at least three 1s?
d) an equal number of 0s and 1s?

18. A coin is flipped eight times where each flip comes up either heads or tails. How many possible outcomes

a) are there in total?
b) contain exactly three heads?
c) contain at least three heads?
d) contain the same number of heads and tails?

40. How many ways are there to select 12 countries in the United Nations to serve on a council if 3 are selected from a block of 45, 4 are selected from a block of 57, and the others are selected from the remaining 69 countries?

44. Find a formula for the number of ways to seat r of n people around a circular table, where seatings are considered the same if every person has the same two neighbors without regard to which side these neighbors are sitting on.

46. How many ways are there for a horse race with four horses to finish if ties are possible? [*Note:* Any number of the four horses may tie.]

Exercises from Section 6.4: (16), (32)

16. The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}$, $0 \le k \le 10$, is:

1 10 45 120 210 252 210 120 45 10 1

Use Pascal's identity to produce the row immediately following this row in Pascal's triangle.

32. Show that if n is a positive integer, then $\binom{2n}{2} = 2\binom{n}{2} + n^2$

a) using a combinatorial argument.
b) by algebraic manipulation.

Exercises from Section 6.5: (10), (32), (34), (54)

10. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

a) a dozen croissants?
b) three dozen croissants?
c) two dozen croissants with at least two of each kind?
d) two dozen croissants with no more than two broccoli croissants?
e) two dozen croissants with at least five chocolate croissants and at least three almond croissants?
f) two dozen croissants with at least one plain croissant, at least two cherry croissants, at least three chocolate croissants, at least one almond croissant, at least two apple croissants, and no more than three broccoli croissants?

32. How many different strings can be made from the letters in *MISSISSIPPI*, using all the letters?

34. How many different strings can be made from the letters in *AARDVARK*, using all the letters, if all three *A*s must be consecutive?

54. How many ways are there to put five temporary employees into four identical offices?