Queueing problem solutions

1

Given: $\lambda = 8/\text{min}, E(L_q) \leq 10$

Find: $E(T_s)$

$$E(T_q) = \frac{E(L_q)}{\lambda} = \frac{10}{8/\text{min}} = 1.25 \text{ min}$$
$$E(T_q) = \frac{\rho}{\mu - \lambda} = \frac{\frac{\lambda}{\mu}}{\mu - \lambda} = 1.25$$
$$1.25\mu^2 - 10\mu - 8 = 0$$
$$\mu = \frac{10 + \sqrt{100 + 40}}{2.5}/\text{min}$$
$$E(T_s) = \frac{1}{\mu} = \frac{2.5}{10 + \sqrt{100 + 40}} \text{ min} \approx 6.87 \text{ sec}$$

2

Given: $E(T) \leq 10 \text{ min}, \lambda = 5/\text{hour} = \frac{1}{12}/\text{min}$

Find: $E(T_s)$

$$E(T_q) = \frac{\rho E(T_s)}{1-\rho}$$
$$E(T) = E(T_q) + E(T_s) = \frac{1}{\mu - \lambda} = 10 \text{ min}$$
$$\mu = \frac{11}{60}/\text{min}$$
$$E(T_s) = \frac{60}{11} \text{ min} \approx 5.45 \text{ min}$$

3

$$E(T)' = E(T_q)' + E(T_s)' = \frac{1}{\mu' - \lambda} = 0.5 \times E(T) = \frac{0.5}{\mu - \lambda}$$
$$\mu' = 2\mu - \lambda = \frac{17}{60}/\text{ min}$$
$$\frac{\frac{17}{60} - \frac{11}{60}}{\frac{11}{60}} = \frac{6}{11} \approx 54.5\%$$

4

Given: $E(L) = 3, \lambda = 3/\text{ hour}$

Find: $E(T_r), E(T)$

$$E(T) = \frac{E(L)}{\lambda} = 1 \text{ hour}$$
$$E(T_r) = E(T_s) = \frac{1}{\mu} = \frac{1}{\frac{1}{E(T)} + \lambda} = \frac{1}{4} \text{ hour} = 15 \text{ min}$$

5

Given: $\lambda = 50/\text{sec}, E(T_s) = 15$ ms

Find: $E(T), E(L_q)$

$$
\begin{aligned}
E(T) &= \frac{1}{\mu - \lambda} = 60 \text{ ms} \\
\rho &= \lambda E(T_s) = 0.75 \\
E(T_q) &= \frac{\rho}{\mu(1-\rho)} = 45 \text{ ms} \\
E(L_q) &= \lambda E(T_q) = 2.25
\end{aligned}
$$

6

Given: $\lambda = 80/\text{sec}, E(T_s) = 7.5$ ms

Find: $E(T)$

$$
E(T) = \frac{1}{\mu - \lambda} = \frac{1}{\frac{1}{E(T_s)} - \lambda} = 18.75 \text{ ms}
$$